A VCG Mechanism for Optimal Social Operation of Shared Batteries in Energy Communities ⋆

David Angeli [*,**] **Davide Martini** [**] **Giacomo Innocenti** [**]
Sabato Manfredi [***] **Leonardo Molino** [***]

[*] *Dept. of Electronic and Electrical Engineering, Imperial College London, U.K.*
[**] *Dept. of Information Engineering, University of Florence, Italy*
[***] *Dept. of Electrical Engineering and Information Technologies, University of Naples Federico II, Italy*

Abstract: We propose a centralised mechanism for the optimal social operation of a shared battery within energy communities of self-interested households, with possibly non-aligned objectives and demand requirements. The mechanism operates in the best interest of the community and collects payments (in addition to the operational electricity charges) so that individual users have a rational incentive to declare the true value of their energy demands. A detailed case study is presented, showing the performance of the scheme and highlighting its merits in situations where the shared battery is too small to cater for the needs of the community.

Keywords: Optimal operation and control of power systems, Smart Grids, Game Theory, Mechanism design, Cyber-Physical Systems

1. INTRODUCTION

Energy communities represent an innovative paradigm where groups of users share energy infrastructures to enhance efficiency, reduce costs, and promote sustainable energy consumption Moroni (2024); Amin et al. (2025); Morstyn and McCulloch (2018). Unlike traditional energy markets that prioritize individual profit maximization, energy communities emphasize equitable energy distribution, cost reduction, and collective benefits. The primary objective is to optimize energy consumption and costs at a societal level rather than generating revenue for the system Amin et al. (2025).

The concept of energy communities has gained significant attention in recent years, with various studies analyzing their potential in promoting decentralized energy management. Research by Barabino et al. (2023) discusses the role of renewable energy sources in energy communities and their economic viability, while van der Schoor and Scholtens (2016) explores the social and technical challenges in implementing such systems. Further, Tushar et al. (2020) emphasizes the importance of advanced optimization mechanisms to ensure efficiency and fairness in resource allocation within energy communities. By leveraging shared infrastructures such as community batteries, these initiatives facilitate demand-side management Parag and Sovacool (2016).

A key feature of energy communities is the ability to trade energy among members, ensuring cost reduction through collective optimization rather than individual financial gains. This approach harmonizes energy availability with demand patterns while effectively utilizing shared storage systems such as community batteries Zhang et al. (2018); Long et al. (2017). Studies such as Sousa et al. (2019) propose blockchain-based solutions for transparent and secure peer-to-peer (P2P) energy trading.

Moreover, energy communities contribute to grid stability by efficiently integrating renewable energy sources and reducing peak demand, thereby fostering sustainability and resilience Amin et al. (2025). The proposed model enables these communities to define pricing mechanisms reflecting internal cost structures and shared benefits rather than being subject to market-driven pricing aimed at profit maximization.

Another significant application is P2P energy trading, where individual users can buy and sell energy under the supervision of a central system manager Sousa et al. (2019). This ensures fair pricing mechanisms by regulating energy prices based on supply and demand while encouraging active user participation in energy management, ultimately optimizing social welfare. Recent work Soto et al. (2021) further highlights how intelligent decision-making frameworks can improve the efficiency and equity of P2P transactions within energy communities.

⋆ This work was supported by project "Resilient Optimization in Distributed Cyber-Physical Control Problems subject to Adversarial Behaviour", funded in part by Italian Ministry of University and Research (MUR) within the Progetti di Rilevante Interesse Nazionale (PRIN) 2022 Program under Project 20225MESZB, and in part by European Union - Next Generation EU, Mission 4 Component 2.

In this scenario we propose an allocation mechanism to address the core operational challenges of energy communities by enabling allocation of shared battery usage based on user preferences and needs; minimize overall social costs (assumed as the sum of the costs of individual users) with fair distribution of energy and associated costs among community members. Since individual costs are private and known only to individual users, it may happen that some malicious users may not to declare the truthful costs to the aim of optimize his own interests. With this in mind, we propose to implement the so called Vickrey-Clarke-Groves (VCG) mechanism Vickrey (1961); Clarke (1971); Groves (1973), since being truthful is a dominant under any Groovs mechanism and due to its property of incentive compatibility Shoham and Leyton-Brown (2008).

More specifically, this paper contributes to the field of energy community optimization by:

1. proposing a social optimization framework for shared battery usage;
2. introducing a fair cost allocation mechanism via a Vickrey-Clarke-Groves (VCG) mechanism, which promotes truthfulness, Vickrey (1961);
3. formulating a general convex optimization model that accommodates elastic and flexible energy consumption preferences.

The proposed work introduces novel elements in dynamic pricing and fair cost allocation in energy communities and battery management, leveraging on VGC mechanisms, widely adopted in engineering applications, Chremos and Malikopoulos (2024).

The paper is organized as follows: Section 2 introduce the problem formulation, while Section 3 shows how the individual households are modeled. Section 4 is devoted to the main results of the paper, which are practically shown in Section 5 with an example of application. Finally, some conclusion end the paper in Section 6.

2. FRAMEWORK AND PROBLEM SET-UP

Consider a set of households $i \in \mathcal{I} := \{1, 2, \ldots, N\}$ which share a large battery and wish to operate in order to reduce social energy prices. Each household has a cost functional which characterizes his energy demand and the cost attributed to excess or lack of energy. This is a convex function $J_i(x)$, where x is a vector of $[x_1, x_2, \ldots, x_Q]$ to characterize energy in-take throughout the day. Typically $J_i(x)$ could be of additive form:

$$J_i(x) = \sum_{t=1}^{Q} J_i^t(x_t)$$

but more general coupled expressions are also allowed, for instance when a certain total power demand at two (consecutive) time-instant is paramount, but the explicit timing is not important, then a term of the type $|x^t + x^{t+1} - E|$ could be present (allowing to model some form of flexible demand). This is general enough to allow users to express very detailed power schedule preferences. Each household i, receives power either directly from the grid, at rate $x_G^{i,t}$ or from the battery, at rate $x_B^{i,t}$. Both are non-negative quantities. The price applied by the grid is pre-assigned, and equal to p^t, while the battery charges individual households λ^t per unit of energy, which is a decision variable of the social optimisation problem. Overall each agent declares its preferences in the form of a function $\hat{J}_i(x)$, and the battery VCG mechanism aims to achieve the social optimum by solving the following problem:

$$\min_{\lambda^t, x_G^{i,t}, x_B^{i,t}, s_t, u_t} \sum_{i=1}^{N} \left(\hat{J}_i(x_G^i + x_B^i) + \sum_{t=1}^{Q} \lambda^t x_B^{i,t} + p^t x_G^{i,t} \right) \tag{1}$$

subject to:

$$s_{t+1} = s_t + u_t - \sum_{i=1}^{N} x_B^{i,t} \qquad t = 1, \ldots, Q$$
$$s_{Q+1} = s_1$$
$$0 \leq s_t \leq E_{\max}$$
$$\sum_{t=1}^{Q} p^t u_t \leq \sum_{t,i} \lambda^t x_B^{i,t}$$
$$-P_{dc} \leq u_t - \sum_{i=1}^{N} x_B^{i,t} \leq P_{ch} \qquad t = 1 \ldots Q$$
$$u_t \geq 0, \qquad x_B^{i,t} \geq 0, \qquad x_G^{i,t} \geq 0,$$

Since λ^t is a decision variable, rather than an exogenous parameter like p^t, dependence of the cost function and constraints is bilinear in λ^t and $x_B^{i,t}$. This significantly aggravates the optimisation problem, therefore, we recommend to leverage on linear programming techniques, the adoption of a unique parameter λ which is not a function of time, viz. $\lambda^t = \lambda$ for all $t = 1, \ldots, Q$. The extension to bidirectional trading with the grid can also be allowed by introducing p_+^t (energy cost when buying from the grid) and p_-^t (selling value of energy to the grid). Normally $0 \leq p_-^t \leq p_+^t$. Similarly u_t can be decomposed into the sum of u_+^t (energy bought from the grid) and u_-^t (energy sold to the grid).

$$\min_{\lambda, x_G^{i,t}, x_B^{i,t}, s_t, u_t} \sum_{i=1}^{N} \left(\hat{J}_i(x_G^i + x_B^i) + \sum_{t=1}^{Q} \lambda x_B^{i,t} + p_+^t x_G^{i,t} \right)$$

subject to:

$$s_{t+1} = s_t + u_+^t - u_-^t - \sum_{i=1}^{N} x_B^{i,t} \qquad t = 1, \ldots, Q$$
$$s_{Q+1} = s_1$$
$$0 \leq s_t \leq E_{\max}$$
$$\sum_{t=1}^{Q} (p_+^t u_+^t - p_-^t u_-^t) \leq \sum_{t,i} \lambda x_B^{i,t}$$
$$-P_{dc} \leq u_+^t - u_-^t - \sum_{i=1}^{N} x_B^{i,t} \leq P_{ch} \qquad t = 1 \ldots Q$$
$$x_B^{i,t} \geq 0, \qquad x_G^{i,t} \geq 0, \qquad u_+^t \geq 0, \qquad u_-^t \geq 0$$

Energy charges to the i-th household, given the optimal solution $\bar{x}_B$, $\bar{x}_G$, $\bar{u}$ and $\bar{\lambda}$, could be computed according to:

$$\sum_{t=1}^{Q} p^t \bar{x}_G^{t,i} + \bar{\lambda} \bar{x}_B^{t,i}.$$

However, this billing strategy is inadequate, as it is likely that the optimisation problem in (1) will have more

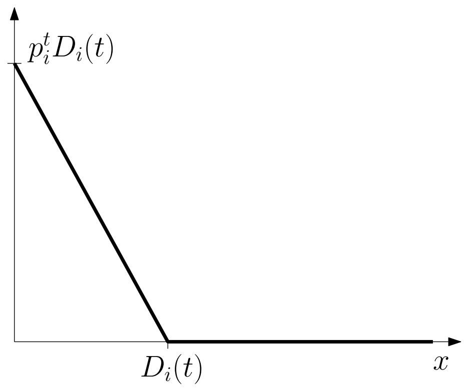

Fig. 1. Demand value at time t for agent i

than one optimal solution, corresponding to different and possibly unfair bills to individual households. To this end, we propose to modify the billing policy according to:

$$\sum_{t=1}^{Q}\left[\frac{\bar{x}_B^{i,t}+\bar{x}_G^{i,t}}{\sum_{i=1}^{N}\bar{x}_B^{i,t}+\bar{x}_G^{i,t}}\sum_{i=1}^{N}(p^t\bar{x}_G^{i,t}+\bar{\lambda}\bar{x}_B^{i,t})\right]. \qquad (2)$$

While this billing policy results in the same total energy cost, it allocates to individual households a share of the total cost of energy at each time t (computed according to $\sum_{i=1}^{N}(p^t\bar{x}_G^{i,t}+\bar{\lambda}\bar{x}_B^{i,t})$) which is proportional to the fraction of demand actually allocated to the household (at that same time). In this respect, it applies a tariff which is independent of how demand in each household is partitioned between battery and grid.

3. MODELING OF INDIVIDUAL HOUSEHOLDS

The aim of this Section is to give more details of how individual users can shape their demand preferences. We proceed under the assumption that each household has an inflexible and a flexible demand profile. For agent i, the inflexible demand profile is a modelled as a function:

$$D_i(t):\{1,2,\ldots,Q\}\to[0,+\infty).$$

We model the flexible demand profile as a quantity of energy E_i^f which the agent needs within a certain time-interval. Of course individual users might have more than one such flexible energy packets, but for simplicity of representation we only consider the case of each household having a single such flexibility option. The interval is in fact any subset $F_i\subset\{1,2,\ldots Q\}$. In order to translate demand profiles into cost functionals, each user needs to decide the value (or cost) that having the demand not served will trigger. For simplicity we adopt linear penalty functions only, but more general options, with progressively increasing stiffness, are also a possibility. The cost functional corresponding to the inflexible demand is expressed as:

$$J_i(x)=\sum_{t=1}^{Q}\max\{-p_i^t(x_t-D_i(t)),0\} \qquad (3)$$

which, at each single time t, corresponds to the function plotted in Fig. 1. Similarly, user i, will consider the following cost functional for flexible demand:

$$J_i^f(x)=\max\left\{-p_i^f\left(\sum_{t\in F_i}[x_t-D_i(t)]-E_i^f\right),0\right\}.$$

Notice that $\sum_{t\in F_i}[x_t-D_i(t)]$ computes the excess of energy intake of the household with respect to its inflexible demand profile across the flexibility time window. The coefficient p_i^f is the value per unit of energy that the user decides for the flexible demand and allows them to evaluate in economic terms the discomfort created by under-delivering across the window F_i, with respect to the target demand level E_i^f anticipated.

4. MECHANISM DESIGN

The contribution of this paper is the design of a finite-horizon VCG mechanism to achieve the social optimum through suitable payment rules which incentivize truthful declaration of households preferences. In the absence of such incentives, users could overestimate their own costs, by declaring extremely high values of parameters p_i^t, essentially hijacking the social optimisation problem previously formulated and operating the battery in their own interest only. In fact, by letting $p_i^t\to+\infty$, satisfaction of the demand of user i at the lowest possible cost achievable through suitable battery operation becomes the goal prioritized by the considered optimisation problem. The mechanism operates by designing a payment rule that penalizes each household i on the basis of the "economical damage" caused by its own preferences to the rest of the community. Below, we denote with barred variables, their optimal values, viz.:

$$(\bar{\lambda},\bar{x}_B^{i,t},\bar{x}_G^{i,t},\bar{s}_t,\bar{u}_t):=\arg\min_{\lambda,x_G^{i,t},x_B^{i,t},s_t,u_t} \qquad (4)$$

$$\sum_{i=1}^{N}\left(\hat{J}_i(x_G^i+x_B^i)+\sum_{t=1}^{Q}\lambda x_B^{i,t}+p^t x_G^{i,t}\right),$$

subject to the same set of constraints previously considered. Notice that optimal decision variables are dependent upon households' declarations of individual costs, $\hat{J}_i(x)$. For each household i, we consider the same form of optimisation problem, without agent i being involved in shaping the control objective. For the sake of consistence with the classical VCG mechanism we keep the associated decision variables, though. We denote its optimal value as $\bar{J}_{-i}$,

$$\bar{J}_{-i}=\min_{\lambda,x_G^{i,t},x_B^{i,t},s_t,u_t}\sum_{j\neq i}\hat{J}_j(x_G^j+x_B^j)+\left(\sum_{t=1}^{Q}\lambda x_B^{j,t}+p^t x_G^{j,t}\right)$$

subject to:

$$s_{t+1}=s_t+u_t-\sum_{j=1}^{N}x_B^{j,t} \qquad t=1,\ldots,Q$$

$$s_{Q+1}=s_1$$

$$0\le s_t\le E_{\max}$$

$$\sum_{t=1}^{Q}p^t u_t\le\sum_{t,j}\lambda x_B^{j,t}$$

$$-P_{dc}\le u_t-\sum_{j=1}^{N}x_B^{j,t}\le P_{ch} \qquad t=1\ldots Q$$

$$u_t\ge 0,\ \max_{t,i}D_i(t)\ge x_B^{j,t}\ge 0,\ \max_{t,i}D_i(t)\ge x_G^{j,t}\ge 0.$$

Notice that an upper-bound to x_B and x_G variables was included. This is not active in the original problem, as all variables are costed, but may improve the performance of

the algorithm as the i-th variables are not included in the cost function for $\bar{J}_{-i}$.
The payment rule, for household i, is postulated as the difference of (declared) cost incurred along the optimal solution $(\bar{\lambda}, \bar{x}_B^{i,t}, \bar{x}_G^{i,t}, \bar{s}_t, \bar{u}_t)$ by the rest of the community, viz. for $j \neq i$, and the minimum (declared) cost experienced by the community if household i costs were not present in the determination of the optimal operational schedule, or equivalently expressed a $\hat{J}_i \equiv 0$ declaration. More precisely, this is computed as:

$$p_i = \sum_{j \neq i} \left(\hat{J}_j(\bar{x}_G^j + \bar{x}_B^j) + \sum_{t=1}^{Q} \bar{\lambda} \bar{x}_B^{j,t} + p^t \bar{x}_G^{j,t} \right) - \bar{J}_{-i}.$$

The above payment rule is appropriate as it re-establishes the compatibility between individual household interests and social optimisation goals. In particular, the total cost experience by agent i as a result of his own, and others' cost functionals:

$$C_i(\hat{J}_i, \hat{J}_{-i}) = J_i(\bar{x}_G + \bar{x}_B) + \sum_{t=1}^{Q} [\bar{\lambda} \bar{x}_B^{i,t} + p^t \bar{x}_G^{i,t}] + p_i.$$

The notation $C_i(\hat{J}_i, \hat{J}_{-i})$ is a game theoretic expression to highlight dependence of C_i from cost declarations of agent i, viz. $\hat{J}_i$, and declarations of the remaining households in the community $\hat{J}_{-i}$. The celebrated VCG truthfulness property is stated below, and is the main reason for proposing this type of scheme within energy communities:

Theorem 1. Under the considered payment rule, truthful cost declarations, viz. $\hat{J}_i(x) \equiv J_i(x)$ are a dominant strategy for agent i, for all alternative declarations $\hat{J}_i$ and for all possible declarations of the remaining community members, $\hat{J}_{-i}$, i.e.:

$$C_i(J_i, \hat{J}_{-i}) \leq C_i(\hat{J}_i, \hat{J}_{-i})$$

Proof. Notice that each household is bound to pay the battery management system, the cost of the energy absorbed, viz. $\lambda(\sum_{t=1}^{Q} x_B^{i,t})$ and paying to the grid $\sum_{t=1}^{Q} p^t x_G^{i,t}$. Overall, each household i, prior to the design of the payment rule p_i, experiences the cost:

$$f_i(x) := J_i(x_G^i + x_B^i) + \lambda(\sum_{t=1}^{Q} x_B^{i,t}) + \sum_{t=1}^{Q} p^t x_G^{i,t}.$$

We remark that optimisation problem (1) is of the form:

$$\min_{x \in \mathbb{X}} \sum_i f_i(x),$$

where x collects all decision variables $[x_G, x_B, u, s, \lambda]$. It is therefore a social optimisation, with respect to individual households costs, where $\mathbb{X}$ is a set modeling the constraints previously formulated. Hence, computing the social minimizers $\bar{x}$ as in (4) implements a classical VCG scheme, by making use of declared costs from all households (which we may denote as $\hat{f}_i(x)$, comprising the declared demand-related discomfort costs $\hat{J}_i$ and the actual operational costs). Accordingly, the indicated payment rule corresponds to:

$$p_i := \sum_{j \neq i} \hat{f}_j(\bar{x}) - \min_{x \in \mathbb{X}} \sum_{j \neq i} \hat{f}_j(x).$$

Let us emphasize dependence of $\bar{x}$ from $\hat{f}_i$ and $\hat{f}_{-i}$, by denoting $\bar{x}(\hat{f}_i, \hat{f}_{-i})$. Taking the point of view of household i we see that:

$$f_i(\bar{x}(f_i, \hat{f}_{-i})) + \sum_{j \neq i} \hat{f}_j(\bar{x}(f_i, \hat{f}_{-i})) \leq f_i(x) + \sum_{j \neq i} \hat{f}_j(x)$$

for any other feasible $x \in \mathbb{X}$. Subtracting $\bar{J}_{-i}$ from both sides of the previous inequality yields:

$$C_i(J_i, \hat{J}_{-i}) \leq f_i(x) + \sum_{j \neq i} \hat{f}_j(x) - \bar{J}_{-i}. \quad (5)$$

In particular, for $x = \bar{x}(\hat{f}_i, \hat{f}_{-i})$ inequality (5) becomes $C_i(J_i, \hat{J}_{-i}) \leq C_i(\hat{J}_i, \hat{J}_{-i})$ (where we exploited that $\bar{J}_{-i}$ is not affected by the declaration of agent i, $\hat{f}_i$).

Such considerations are not affected by the adoption of the alternative billing rule (2). In fact, defining $\tilde{f}_i(x)$ as:

$$\tilde{f}_i(x) = J_i(x_G^i + x_B^i) + \sum_{t=1}^{Q} \frac{x_B^{i,t} + x_G^{i,t}}{\sum_{i=1}^{N} x_B^{i,t} + x_G^{i,t}} \sum_{i=1}^{N} (\lambda x_B^{i,t} + p^t x_G^{i,t}),$$

we see that $\sum_{i=1}^{N} \tilde{f}_i(x) = \sum_{i=1}^{N} f_i(x)$. In particular then, the optimal solution $\bar{x}$ is unaffected (for the same cost declarations) by this modified billing rule. The cost experienced by household i can be expressed as $\tilde{C}_i(\hat{J}_i, \hat{J}_{-i})$:

$$J_i(\bar{x}_G + \bar{x}_B) + \sum_{t=1}^{Q} \left[\frac{\hat{x}_B^{i,t} + \hat{x}_G^{i,t}}{\sum_{i=1}^{N} \hat{x}_B^{i,t} + \hat{x}_G^{i,t}} \sum_{i=1}^{N} (p^t \hat{x}_G^{i,t} + \hat{\lambda} \hat{x}_B^{i,t}) \right] + \tilde{p}_i,$$

where:

$$\tilde{p}_i = \sum_{j \neq i} \hat{J}_j(\bar{x}_G + \bar{x}_B)$$

$$+ \sum_{j \neq i} \sum_{t=1}^{Q} \left[\frac{\hat{x}_B^{j,t} + \hat{x}_G^{j,t}}{\sum_{j=1}^{N} \hat{x}_B^{j,t} + \hat{x}_G^{j,t}} \sum_{j=1}^{N} (p^t \hat{x}_G^{j,t} + \hat{\lambda} \hat{x}_B^{j,t}) \right] - \bar{J}_{-i}.$$

Overall, then $\tilde{C}_i(\hat{J}_i, \hat{J}_{-i}) = C_i(\hat{J}_i, \hat{J}_{-i})$ and truthful declarations are dominant strategies also according to this billing policy.

5. EXAMPLE OF APPLICATION

As an example of application, we consider the case of $N = 6$ households which share a battery of $E_{max} = 50$ kWh and require the power demand profile during the day displayed in Fig. 2. Since the community wishes to operate the battery to the aim of reducing the social energy price, agents are called upon to communicate their (private) energy demand (measured in kW) and associated value of service to the mechanism (measured in euro cents). We consider first the case of truthful agents, i.e. each agent has complete knowledge of their private costs and faithfully discloses them to the mechanism. In Fig. 3 it is reported the agent's penalty associated to the lack of energy and the grid prices over the day.

From (1) it can be noticed that, in order to simplify the minimization problem as a sequence of linear programming (LP) problems, we necessarily solve it for $\lambda^t = \lambda$ constant in time, with λ within the compact interval $[\min_t p^t, \max_t p^t]$. With this approach, the optimal value of $\lambda = \bar{\lambda}$ is the one resulting from the minimization of the social cost function. Furthermore, since to implement

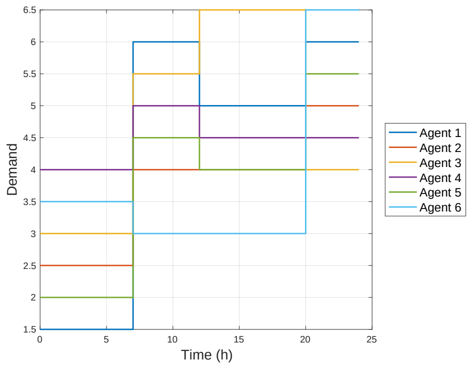

Fig. 2. Agents' power demand profile (kW) during the day.

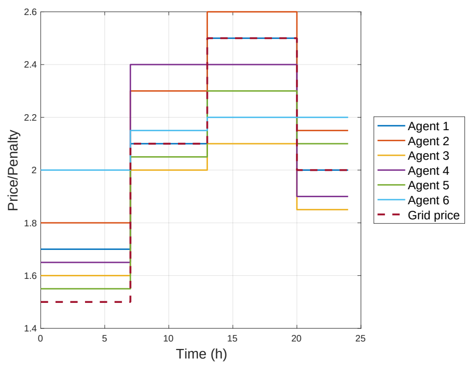

Fig. 3. Agents' (private) penalty (in euro cents/kW) associated to the lack of energy and grid prices.

directly equation (3) for the inflexible demand would introduce a nonlinearity inside the cost function, the latter is handled by making use of an auxiliary variable $z_{i,t} \geq 0$, with $J_i(x) = \sum_{t=1}^{24} z_{i,t}$, and additional constraints $-p_i^t(x_B^{i,t} + x_G^{i,t} - D_i(t)) \leq z_{i,t}$. Therefore, for each value of λ, the minimization problem (1) assumes the following form

$$\min_{x_G^{i,t}, x_B^{i,t}, s_t, u_t} \sum_{i=1}^{6} \left(\sum_{t=1}^{24} z_{i,t} + \lambda x_B^{i,t} + p^t x_G^{i,t} \right) \tag{6}$$

subject to:

$$s_{t+1} = s_t + u_t - \sum_{i=1}^{6} x_B^{i,t} \qquad t = 1, \ldots, 24$$

$$s_{25} = s_1$$

$$0 \leq s_t \leq 50$$

$$\sum_{t=1}^{24} p^t u_t \leq \sum_{t,i} \lambda x_B^{i,t}$$

$$0 \leq z_{i,t} \qquad - p_i^t(x_B^{i,t} + x_G^{i,t} - D_i(t)) \leq z_{i,t}$$

$$-12.5 \leq u_t - \sum_{i=1}^{6} x_B^{i,t} \leq 12.5 \qquad t = 1, \ldots, 24$$

$$u_t \geq 0, \ \max_{t,i} D_i(t) \geq x_B^{j,t} \geq 0, \ \max_{t,i} D_i(t) \geq x_G^{j,t} \geq 0.$$

where $P_{dc} = P_{ch} = E_{max}/4$ meaning that the battery takes a minimum of 4 hours to charge and/or discharge. Solving it with the LP MATLAB toolbox, we obtain the trend reported in Fig. 4 and Fig. 5. A sufficiently "large" battery with adequately large power ratings, would allow to transfer energy bought at the minimum price from the grid to all households. If the agent's penalty associated to energy demand at time t is less then the minimum of the grid price over the day, then that demand will not be served. Conversely, if the penalty is higher than the grid price (at the same time) it will be served no matter what (e.g. agent 2 between 14 and 20 o'clock). Finally, if the agent i's energy penalty is between the minimum and the current value of the grid energy price, the energy demand might be served or not depending on the particular situation and the battery operating schedule. Overall, the mechanism aims to buy and to deliver energy at minimum price while optimising the overall social costs (viz. the sum of energy bills and penalties experienced by lack of delivered energy). To validate the approach, notice

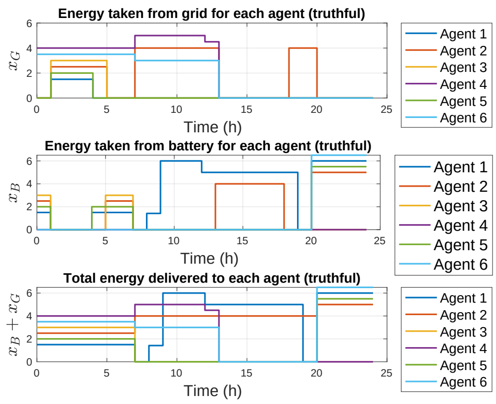

Fig. 4. Energy (kW) provided to the agents during the day.

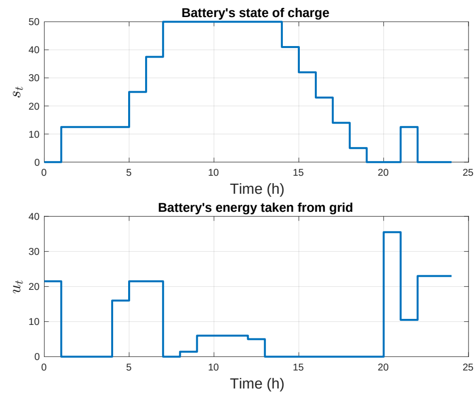

Fig. 5. Battery's state of charge (kW) and absorbed energy (kW) from grid during the day.

that agent's 3 truthful reporting implies that his demand between 14 and 20 o'clock is not served. Suppose that, in the face of this, agent 3 declares false by increasing the penalty associated with his demand in the corresponding time slot changing his declaration from a value of 2.1 to 4 euro cents. With this untruthful declaration the resulting schedule is reported in Fig. 6, where agent 3's energy demand is now served at the expense of the other agents. However, thanks to the VCG payment compensations, his overall costs (comprising energy bills, VCG payments and penalties associated to lack of service) are higher than before, as it is shown in Fig. 7. Fig. 8 shows the variation of agent 3's costs as his declaration changes (with respect to the scalar penalty parameter in the 14 to 20 time range), from which it is clear that a dominant strategy for agent 3 is to declare its real costs (2.1 is a minimum of the costs considered, albeit fairly flat).

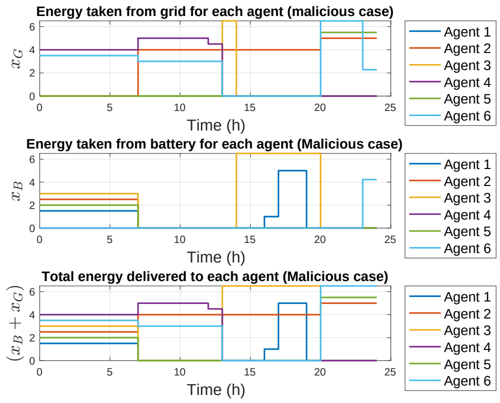

Fig. 6. Energy (kW) provided to the agents during the day in the malicious case.

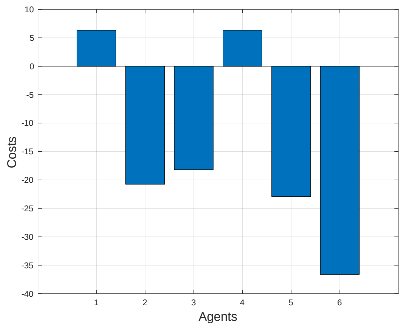

Fig. 7. Difference of agents' costs (in euro cents) between the truth case and the malicious case.

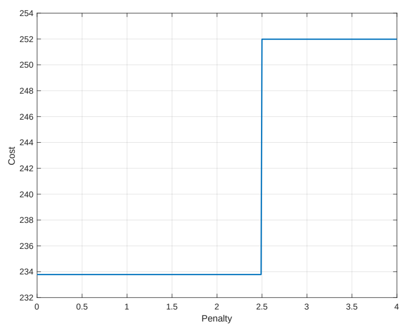

Fig. 8. Trend of agent 3's costs (in euro cents) as his declaration varies.

6. CONCLUSIONS

This paper deals with the design of optimal social operation of a shared battery in the presence of self-interested households, that may wish to steer battery operations according to their own interests at the expense of other users. By implementing a VCG-scheme of payments, each user is encouraged to truthfully disclose their private value of serviced energy to the mechanism. An example of application with 6 users has been tackled showing that even if the battery is too small to deliver cheap power to all users and the socially optimal solution does not deliver energy to a specific user, the dominant strategy for each household still remains the truthful declaration.

REFERENCES

Amin, M.A., Procopio, R., Invernizzi, M., Bonfiglio, A., and Jia, Y. (2025). Exploring the role of energy communities: A comprehensive review. *Energy Conversion and Management: X*, 25, 100883. doi: https://doi.org/10.1016/j.ecmx.2025.100883.

Barabino, E., Fioriti, D., Guerrazzi, E., Mariuzzo, I., Poli, D., Raugi, M., Razaei, E., Schito, E., and Thomopulos, D. (2023). Energy communities: A review on trends, energy system modelling, business models, and optimisation objectives. *Sustainable Energy, Grids and Networks*, 36, 101187. doi: https://doi.org/10.1016/j.segan.2023.101187.

Chremos, I.V. and Malikopoulos, A.A. (2024). Mechanism design theory in control engineering: A tutorial and overview of applications in communication, power grid, transportation, and security systems. *IEEE Control Systems Magazine*, 44(1), 20–45.

Clarke, E.H. (1971). Multipart pricing of public goods. *Public Choice*, 11, 17–33.

Groves, T. (1973). Incentives in teams. *Econometrica*, 41(4), 617–631.

Long, C., Wu, J., Zhang, C., and Jenkins, N. (2017). Peer-to-peer energy sharing through a two-stage aggregated battery control model. *Applied Energy*, 226, 276–288.

Moroni, S. (2024). Energy communities, distributed generation, renewable sources: Close relatives or potential friends? *Energy Research & Social Science*, 118, 103828. doi:https://doi.org/10.1016/j.erss.2024.103828.

Morstyn, T. and McCulloch, M. (2018). Using peer-to-peer energy-trading platforms to incentivize prosumers to form federated power plants. *Nature Energy*, 3, 94–101.

Parag, Y. and Sovacool, B. (2016). Electricity market design for the prosumer era. *Nature Energy*, 1, 16032.

Shoham, Y. and Leyton-Brown, K. (2008). *Multiagent systems: Algorithmic, game-theoretic, and logical foundations*. Cambridge University Press.

Soto, E.A., Bosman, L.B., Wollega, E., and Leon-Salas, W.D. (2021). Peer-to-peer energy trading: A review of the literature. *Applied Energy*, 283, 116268. doi: https://doi.org/10.1016/j.apenergy.2020.116268.

Sousa, T., Soares, T., Pinson, P., Moret, F., Baroche, T., and Sorin, E. (2019). Peer-to-peer and community-based markets: A comprehensive review. *Renewable and Sustainable Energy Reviews*, 104, 367–378. doi: https://doi.org/10.1016/j.rser.2019.01.036.

Tushar, W., Saha, T.K., Yuen, C., Smith, D., and Poor, H.V. (2020). Peer-to-peer trading in electricity networks: An overview. *IEEE Transactions on Smart Grid*, 11(4), 3185–3200. doi:10.1109/TSG.2020.2969657.

van der Schoor, T. and Scholtens, B. (2016). Community energy: a critical review of the literature.

Vickrey, W. (1961). Counterspeculation, auctions, and competitive sealed tenders. *J. of Finance*, 16(1), 8–37.

Zhang, C., Wu, J., Long, C., and Cheng, M. (2018). Peer-to-peer energy trading in a microgrid. *Applied Energy*, 220, 1–12.